Signals and Systems
E-623

Lecture 5
Systems Properties
and Fourier Series Analysis

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Linear Systems (1)

- A **linear system** exhibits the **additivity** property:

$$x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2 \qquad x_1 + x_2 \longrightarrow y_1 + y_2$$

- It also must satisfy the **homogeneity** or **scaling** property:

$$x \longrightarrow y \qquad kx \longrightarrow ky$$

- These can be combined into the property of **superposition**:

$$x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2 \qquad k_1 x_1 + k_2 x_2 \longrightarrow k_1 y_1 + k_2 y_2$$

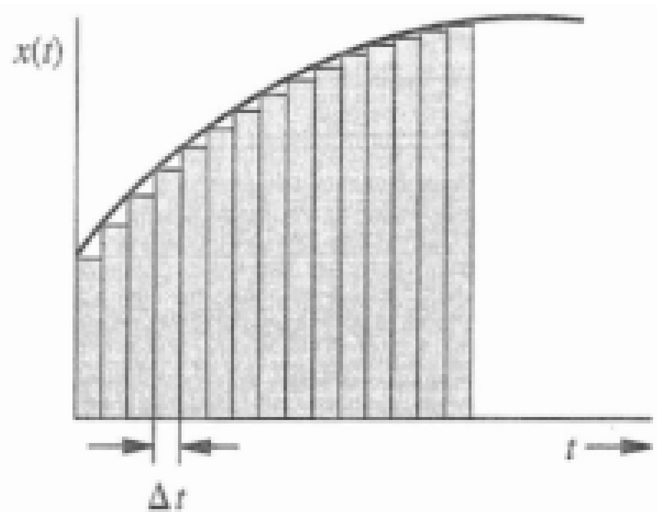
- A non-linear system is one that is NOT linear (i.e. does not obey the principle of superposition)

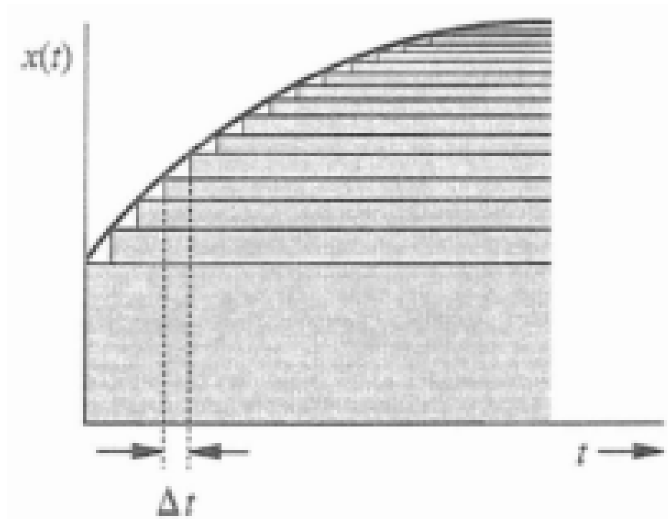
LINEAR AND NONLINEAR SYSTEMS

- Many systems are nonlinear. For example: many circuit elements (e.g., diodes), dynamics of aircraft, econometric models,…
- However, we focus exclusively on **linear** systems.
- Why?
  - Linear models represent accurate representations of behavior of many systems (e.g., linear resistors, capacitors, other examples given previously,…)
  - Can often linearize models to examine "small signal" perturbations around "operating points"
  - Linear systems are analytically tractable, providing basis for important tools and considerable insight

Linear Systems (5)

- Almost all systems become **nonlinear** when large enough signals are applied
- Nonlinear systems can be **approximated** by linear systems for **small-signal analysis** – greatly simply the problem
- Once superposition applies, analyse system by decomposition into zero-input and zero -state components
- Equally important, we can represent $x(t)$ as a sum of simpler functions (pulse or step)





$$x(t) = a_1x_1(t) + a_2x_2(t) + \cdots + a_mx_m(t)$$

$$y(t) = a_1y_1(t) + a_2y_2(t) + \cdots + a_my_m(t)$$

L1.7-1 p

TIME-INVARIANCE (TI)

Informally, a system is time-invariant (TI) if its behavior does not depend on what time it is.

- Mathematically (in DT): A system $x[n] \rightarrow y[n]$ is TI if for <u>any</u> input $x[n]$ and <u>any</u> time shift n_0,

If $\quad x[n] \rightarrow y[n]$

then $\quad x[n - n_0] \rightarrow y[n - n_0]$.

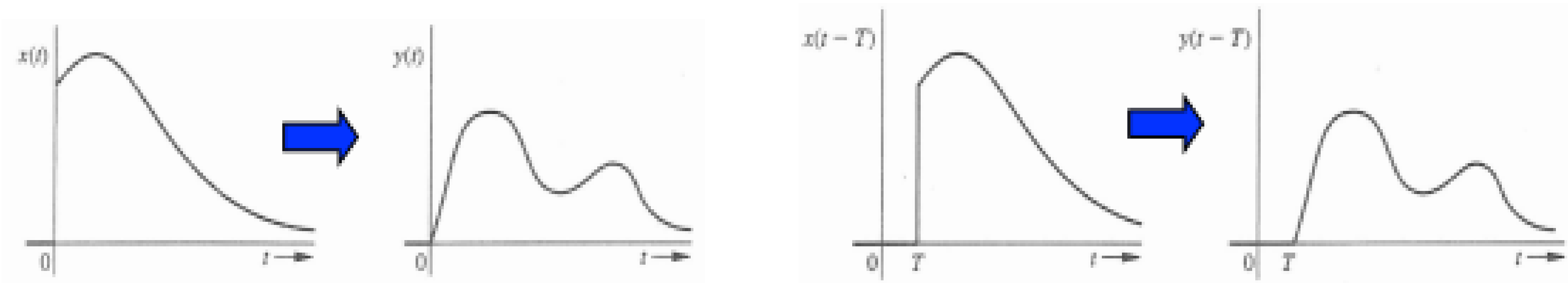
- Similarly for a CT time-invariant system,

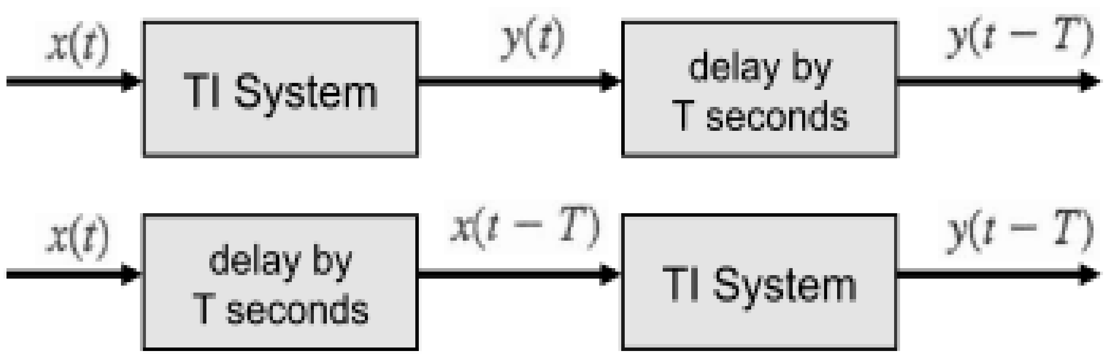
If $\quad x(t) \rightarrow y(t)$

then $\quad x(t - t_o) \rightarrow y(t - t_o)$.

Time-Invariant Systems

- **Time-invariant system** is one whose parameters do not change with time:





CAUSALITY

- A system is causal if the output does not anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time.
- All real-time physical systems are causal, because time only moves forward. Effect occurs after cause. (Imagine if you own a noncausal system whose output depends on tomorrow's stock price.)
- Causality does not apply to spatially varying signals. (We can move both left and right, up and down.)
- Causality does not apply to systems processing recorded signals, e.g. taped sports games vs. live broadcast.

CAUSAL OR NONCAUSAL

$y(t) = x^2(t-1)$

E.g. $y(5)$ depends on $x(4)$ ... causal

$y(t) = x(t+1)$

E.g. $y(5) = x(6), y$ depends on future $\Rightarrow$ noncausal

$y[n] = x[-n]$

E.g. $y[5] = x[-5]$ ok, but
$y[-5] = x[5], y$ depends on future $\Rightarrow$ noncausal

$y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$

E.g. $y[5]$ depends on $x[4]$ ... causal

Invertible and Noninvertible Systems

- Let a system S produces $y(t)$ with input $x(t)$, if there exists another system S_i, which produces $x(t)$ from $y(t)$, then S is invertible
- Essential that there is **one-to-one mapping** between input and output
- For example if S is an amplifier with gain G, it is invertible and S_i is an attenuator with gain 1/G
- Apply S_i following S gives an **identity system** (i.e. input $x(t)$ is not changed)